Jelena Stefanović-Marinović 1
Marko Petković 2
Ivan Stanimirović 3
Miloš Milovančević 4

A MODEL OF PLANETARY GEAR MULTICRITERIA OPTIMIZATION

Summary

The application of multicriteria optimization to planetary gear train is the objective in this paper. A model of planetary gear multicriteria optimization based on an original algorithm is created.

The basis of the algorithm are experimentally determined approximations of analytical expressions for volume, mass, efficiency and production costs. The following is adopted as optimization variables: teeth numbers, number of planets, module and facewidth. Conditions required for the proper functioning of the system in the scope of geometry and strength are expressed by the functional constraints.

Apart from the determination of the set of Pareto optimal solutions, methods for choosing the optimal solution from this set are included in the mathematical model. The complete optimization procedure is implemented in *PlanGears* software. Based on numerical examples obtained by the application of this software, the comparison of the optimization methods and program results analysis is presented.

Key words: planetary gear train, multicriteria optimization, mathematical model, Pareto optimal solutions

1. Introduction

The multi-objective optimization is a method which is implemented in the development of many products and processes in various ways. The number and actuality of published researche indicate the importance and contemporaneity of optimization topics.

Multicriteria optimization problems are very common in many scientific and technical solutions. Optimization of gear trains as concrete technical systems supposes a very complex mathematical model which has to describe the operation of a real system in real circumstances.

Planetary gear trains are an important kind of gear transmissions and they also can be the subject of multicriteria optimization. It is impossible to include every type of these transmissions in the same paper, especially taking into consideration that they have their own geometrical conditions and can also be one-stage and multi-stage.

In the available literature, there are not many papers about the application of multicriteria optimization on gear transmissions, especially on planetary gear transmissions.

In reference [1], design problems of gears with minimal dimensions are indicated. A population-based evolutionary multi-objective optimization approach, based on the concept of Pareto optimality, is proposed in paper [2] in order to design helical gears (minimize both the mass of the gearing and the flank adhesive wear speed). Paper [3] presents the choice of the best parameters optimization for obtaining the required gear quality and the optimization of the designing process itself. An analytical and computer aided procedure for the multicriteria design optimization of multistage gear transmission is presented in paper [4].

A simple, descriptive and easy-to-handle method for investigating the transmission ratio, the internal power flows and efficiency of complex multiplanetary gearings is introduced in [5]. The process of planetary gear transmission optimization is shown in paper [6] as a method which leads to optimum (housing diameter and gear volume are considered in order to achieve their minimum). A possible model for finding the solution to this problem is the application of stochastic methods, where parameter values vary by accidental numbers.

This paper provides an optimization of the basic type of planetary gear train. The presented approach is based on the original algorithm which is the basis of the mathematical model. The optimization of planetary gear transmission is conducted using four criteria, including the economic criterion, which makes this optimization task a sort of techno-economical optimization.

In order to make a different approach than the models in which an optimal solution is adopted by analising the solutions from the set of Pareto solutions [4,6], the application of multicriteria optimization methods for choosing an optimal solution from Pareto solutions is included in this paper, as well as their comparison. Several multicriteria optimization methods are briefly decribed and applied to planetary gear transmissions.

2.   Mathematical model for planetary gear train optimization

The basic type of planetary gear train (PGT), i.e. a design which has a central sun gear (external gearing - 1), central ring gear (internal gearing - 3), planets (satellites - 2) and carrier (h), shown in Fig.1, is the subject of the paper, limited to geared pairs. Planets are simultaneously in contact with the sun gear and ring gear.

This type of PGT is often used as single stage transmission, so as a building block for higher compound planetary gear trains.

Its advantage according to other PGT types lies, first of all, in its efficiency. The efficiency value varies negligibly in all range of internal gear ratio ($p = |z_3|/z_1$). Also, this type has small overall dimensions and mass and its production costs are relatively low because of the relatively simplified production.

Because of its characteristics, it is applicable in transport and stationary machines without limitation in power and velocity, for example in the responsible parts of the helicopter, caterpillar, mining, agricultural and other machines and so on.

An optimization task is defined by the variables, objective functions and conditions required for the proper functioning of a system determined by the functional constraints.
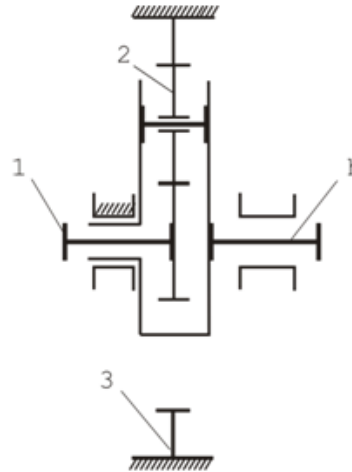
Fig.1 Basic type of planetary gear train (1- sun gear; 2 – planet; 3 – ring gear; h – carrier)

2.1 Variables

In the scope of the mathematical model definition, it is necessary to determine the variables since each objective function is the function of several parameters.

In this paper, the following variables are considered: teeth number of central sun gear z_1, teeth number of planets z_2, teeth number of ring gear z_3, number of planets n_w, gear module m_n and facewidth b.

The optimization variables are of mixed type: numbers of gear teeth (z_1, z_2, z_3) are integers, positive and negative, number of planets (n_w) is a discrete value, module (m_n) is a discrete standard value (acc. to DIN 780), while facewidth (b) is a continual variable. Numbers of gear teeth and number of planets are non-dimensional values, while module and gear width are given in millimeters.

The facewidth as a variable is introduced due to the ratio of pinion facewidth to pinion reference diameter.

In the case where $z_1 \leq z_2$ the the range of the parameter $\psi = b / d_1$, is defined according to the ratio $p = |z_3| / z_1$, $\psi_{bd1min} = p \cdot 0.1$ and $\psi_{bd1max} = p \cdot 0.18$ (if $\psi_{bd1max} > 0.75$, it is necessary to adopt limit value). If the relation of $z_1 > z_2$ exists, the parameter ψ is introduced as $\psi = b / d_2$ [7].

In this way it is possible to vary the facewidth, wile gear teeth have the same values. This procedure is implemented in the developed software.

The fact that for gear ratio $i < 4$ the planet is smaller than the sun gear , i.e. $z_1 > z_2$, is taken into account. First of all, this fact had to be considered due to introducing the facewidth and gear module as variables.

2.2 Objective functions

In this model, the following characteristics are chosen for objective functions of a planetary gear train: volume, mass, efficiency and production cost of gear pairs.

2.2.1 Volume

The volume of gear pairs is used as overall dimensions expression and the approximation of gear by cylinder volume with diameter equal to pitch diameter and height

equal to facewidth. The fact that planets are inside the ring gear makes it possible for the gear volume to be expressed by:

$$V = \frac{\pi}{4} \cdot b \cdot \left(\frac{m_n \cdot z_3}{\cos \beta} \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt23}} \right)^2 \qquad (1)$$

where α_t is the the transverse pressure angle, α_{wt23} is the working transverse pressure angle for the pair 2-3 and β is the helix angle at pitch diameter.

2.2.2   Mass

Mass is determined as sum of all gear masses in transmission. Since the mass of a particular gear is determined as gear volume multiplied by the density of gear material, $m_z = \rho \cdot V_z$, the final expression of this function is:

$$m = 0.25 \cdot \pi \cdot b \cdot \rho \cdot \frac{m_n^2}{\cos^2 \beta} \cdot \left[k_1 \cdot z_1^2 \cdot \frac{\cos^2 \alpha_t}{\cos^2 \alpha_{wt12}} + n_w \cdot k_2 \cdot z_2^2 \cdot \frac{\cos^2 \alpha_t}{\cos^2 \alpha_{wt12}} + k_3 \cdot z_3^2 \cdot \frac{\cos^2 \alpha_t}{\cos^2 \alpha_{wt23}} \right] \qquad (2)$$

To determine the gear mass, the factor of deviation of real gear shape from cylinder (k) has to be taken into account also. For purposes of optimization, i.e. the comparison of gears with different parameters, this factor does not have a great significance, since it is given in advance due to the hub gear form and it is a constant in the process of optimization.

2.2.3   Efficiency

This is one of the most important criteria for the design and evaluation of construction quality. Power losses in planetary transmissions consist of losses in gears contact, losses in bearings and losses due to oil viscosity. The calculation of gear transmissions efficiency is generally confined to losses depending on friction on tooth sides, i.e. on calculation of contact power losses [8,9,10].

According to previous efficiency remarks and having in mind the fact that the subject of optimization is is limited to gear pairs, we consider the following expression for efficiency [10]:

$$\eta = 1 - \frac{z_3}{z_3 - z_1} \left[\frac{0.15}{z_1} + \frac{0.35}{z_2} + \frac{0.20}{z_3} \right] \qquad (3)$$

2.2.4   Production cost

Economic demands must also be taken into consideration in techno-economical optimization. First, these demands are related to *production costs*. These costs consist of production material and the production process costs. The time needed for the production of gears is taken as a measure of production costs and as an economic factor. This function is then determined as a sum of time periods needed for the production of central sun gear (T_1), planets (T_2) and ring gear (T_3), i.e.

$$F_T = T_1 + n_w \cdot T_2 + T_3 \qquad (4)$$

Production times are determined according to the technologies of Fette [11], Lorenc [12] and Höfler [13].

2.3 Functional constraints

Planetary gear trains represent a specific group of gear trains. Therefore, there are numerous exceptions that need to be taken into account for these transmissions to function

correctly compared with classical gear transmissions. The exceptions presented in this article are related to mounting conditions, geometrical conditions and strength conditions.

The mounting conditions comprise the condition of coaxiality, the condition of adjacency and the condition of conjunction [14].

Geometrical conditions relate to undercutting and profile interference, ratio of pressure angle to working transverse pressure angle, tooth thickness and space width, transverse contact ratio value, sliding speeds, ratio of pinion facewidth to pinion reference diameter, etc. These conditions are ensured in accordance with the actual standards (ISO TC 60 list of standards 090915).

The strength conditions, safety factors for bending strength and surface durability of each gear, are provided. For the calculation of load and stress distribution of externally toothed pairs of gears, efficient techniques have been developed, among them the standards for basic calculation DIN 3990 and ISO 6336. The special characteristics of internal toothing, however, are only incompletely taken into account within them. In order to take into consideration the complex connection of internal toothing in planetary gear trains, guideline VDI 2737 has been developed in form of short-term applicability [15]. In this work, safety factors for bending strength and surface durability of each gear are provided according to ISO 6336-1 to ISO 6336-3 [16].

Explicit constraints related to the selection of teeth numbers (z_1, z_2, z_3), planets number n_w and standard values for module m_n are included in calculation procedures.

2.4 Optimization procedure

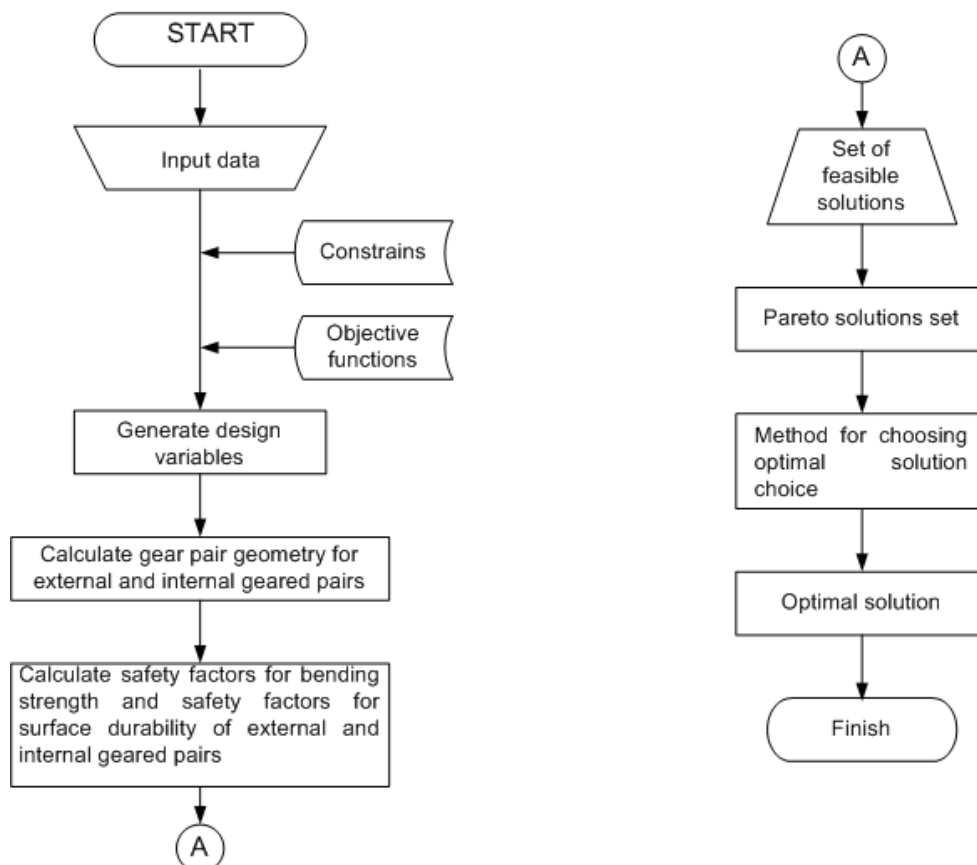The shortened algorithm for the complete optimization procedure is shown in Fig. 2.

Fig.2 Shortened algorithm for optimization procedure

For the given input data (input number of revolution, input torque, service life in hours, application factor, accuracy grade (Q-DIN3961), minimal safety factor - flank, minimal safety factor - root, gears materials, allowed deviation of gear ratio, range of z_1 variation), all 6-tuples of design parameters $(z_1, z_2, z_3, n_w, m_n, b)$ satisfying the functional constraints are generated and the values of the objective functions for every 6-tuple are computed (marked with a capital letter "A" and number).The set of feasible solutions is obtained.

In this generation it is necessary to determine the the profile shift coefficients and the value of the mesh load factor.

The profile shift coefficients are calculated for each planetary gear train determined by variables using the program module which is a part of the PlanGears software. The center distance is calculated using variables and rounded to standard value. The next step is the calculation of profile shift coefficients sum for external gearing and distribution of that sum. In this numerical examples, the distribution according to MAAG is used. At the end of this module, the ring gear profile shift coefficient is calculated.

The value of the mesh load factor K_γ is adopted as the function of planets number. There is a program module for mesh load factor choice for planetary gear train determined by variables and the existence of methods (actions) for equalizing distribution of the load between meshes for multiple paths.

Next, it is necessary to choose only one optimal solution among all the generated solutions. Methods for solving the multicriteria optimization problems are presented in the next section. The complete optimization procedure is implemented in the newly developed *PlanGears* software. The software is written in the programming language Delphi 7.0 and contains a complete 39 step procedure.

3. Multicriteria optimization

In most of optimization problems, several functions which need to be optimized are considered, but they cannot all have optimal values at the same time. Such problems are called non-trivial *multiple criteria* (or *multiple objective*, *multicriteria*) optimization problems [17]. Several methods for solving multicriteria optimization problems are formulated in this section. These formulations are adapted so that they can be directly applied to the planetary gear trains design.

The mathematical model of nonlinear multicriteria problem can be formulated as follows:

$$\max \{ f_1(x), f_2(x), ..., f_k(x) \}$$
$$\text{subject to } x \in S \tag{5}$$

Functions $f_1(x), \ldots, f_k(x)$ are *objective functions* and $x = (x_1, ..., x_n)$ is vector of *decision variables*. These variables must satisfy given constraints which are expressed as inclusion $x \in S$ where S is the set of *feasible solutions* (or *feasible set*). The notation "max" means the simultaneous maximization of all the objective functions. If some objective function needs to be minimized a simple fact that minimization of the function $f_i(x)$ is equivalent to the maximization of the function $-f_i(x)$ can be used. Every point $x \in S$ is mapped to the point $(f_1(x), f_2(x), ..., f_k(x))$ in k - dimensional objective space. Therefore, it can be introduced as the objective set:

$$F = \{(f_1(x), f_2(x), ..., f_k(x)) \mid x \in S\} \tag{6}$$

In the formulated problem, six variables exist, corresponding to the basic design parameters: $x = (x_1, x_2, x_3, x_4, x_5, x_6) = (z_1, z_2, z_3, n_w, m_n, b)$. Also, four objective functions equal to the *volume* ($V(x)$), *mass* ($m(x)$), *efficiency* ($\eta(x)$) and *production costs* ($T(x)$) exist. Since mass, volume and production costs should be minimized, and efficiency should be maximized, the following is denoted in this model:

$$f_1(x) = -V(x), \ f_2(x) = -m(x),$$
$$f_3(x) = \eta_p(x), \ f_4(x) = -T(x) \tag{7}$$

Furthermore, the set S will be defined as the set of all 6-tuples of design parameters such that functional constraints (subsection 2.3) are satisfied. According to the procedure described in Fig. 2, there are finitely many feasible solutions. Hence, the considered multicriteria optimization problem (5) is discrete. All methods that will be applied require a single pass or several (but constant) passes through the feasible set S. Hence the implementation does not suffer from local maximum (minimum) trapping.

It is often useful to know the best possible values for each objective function. These values form a so-called *ideal point* $f^* = (f_1^*, \ldots, f_k^*)$ in the objective space. Its components are computed as

$$f_i^* = \max_{x \in S} f_i(x), \ \text{ for all } \ i = 1, \ldots, k. \tag{8}$$

As it can be seen from the definition, multicriteria optimization problems are mathematically ill-defined. This means that they have a set of mathematically "equally good" optimal solutions in the objective space. The most important criterion for selecting these "equally good" solutions is *Pareto optimality concept*:

Solution $x \in S$ is Pareto optimal if there is no solution $y \in S$ such that holds $f_i(x) \leq f_i(y)$ for all $i = 1, \ldots, n$ and for at least one index i holds strict inequality, i.e. $f_i(x) < f_i(y)$.

Thus, some additional information is needed in order to be able to select one of them as the final solution. This final decision is usually made either by decision maker (human expert) or by the corresponding *scalarized problem*. In the latter case, one or more single criterion optimization scalarized problems have to be constructed and solved. Several methods based on the construction of scalarized problem are presented next.

3.1 Weighted coefficients method

In this method the following scalarized problem is constructed:

$$\max f^M(x) = w_1 f_1^0(x) + \ldots + w_m f_m^0(x)$$
$$\text{s.t. } x \in S \tag{9}$$

Here, weighted coefficients (or weights) w_i are positive real numbers and $f_i^0(x) = (f_i^0)^{-1} f_i(x)$ are normalized objective functions where f_i^0 are normalizing coefficients. In this approach, the components of ideal point $f^* = (f_1^*, f_2^*, f_3^*, f_4^*)$ are used as normalizing coefficients, i.e. $f_i^0 = f_i^*$ for $i = 1, 2, 3, 4$. Therefore, absolute values of all objective functions are between 0 and 1, which simplifies the choice of the weighted coefficients. All solutions obtained by this method are Pareto optimal.

3.2 Lexicographic method

Assuming that objective functions are sorted by the given priorities, it can be observed that if $f_{k_1}(x)$ has maximum priority, then $f_{k_2}(x)$, etc. and $f_{k_m}(x)$ has the least priority. Then, the following list of scalarized problems for $i = 1, \ldots, m$ can be solved:

$$f_{k_i}^{opt} = \max f_{k_i}(x)$$

$$\text{s.t. } x \in S \tag{10}$$

$$f_{k_j}(x) = f_{k_j}^{opt}, \text{ for } j = 1, \ldots, i-1$$

Thus, objective functions are maximized sequentially, and the feasible set is reduced by iterations to the set of optimal solutions in previous iteration. The solution obtained by this method is also Pareto optimal.

3.3 ε- constraints method

In the ε- constraints method, one objective function $f_q(x)$ has to be maximized under the primary and additional conditions is chosen. These additional conditions are in the form of [18] $f_i(x) \geq \varepsilon_i$, where ε_i, $i \neq q$ are given thresholds. Therefore, the following scalarized problem is solved:

$$\max f_q(x)$$

$$\text{s.t. } x \in S \tag{11}$$

$$f_i(x) \geq \varepsilon_i, \text{ for } i = 1, \ldots, n, \ i \neq q$$

In this case, a systematic variation of ε_i the set of Pareto optimal solutions is created [19]. This method is commonly used because it is possible to exactly control the values of all objectives, which is also very important in practical applications.

3.4 Distance method

The main idea in this method is the minimization of distance between one given (infeasible) reference point f^z and the objective set F. The following scalarized problem can be formulated in this way:

$$\min d(f(x), f^z)$$

$$\text{s.t. } x \in S \tag{12}$$

Here $d(x, y)$ can be any metric function. In this implementation, Euclidean distance is used, thus distance obtains the shape:

$$d(x, y) = \sqrt{\sum_{i=1}^{n} w_i (x_i - y_i)^2}$$

Here w_i are given positive real numbers, weight coefficients. Usually, an ideal point f^* is used as a reference point. Under the condition $w_i > 0$ for all $i = 1, \ldots, n$, it can be proven that the solution obtained by this method, using Euclidean metric, is Pareto optimal.

4. Numerical examples

In this section, for gear ratio i in recommended range, the number of 6-tuples $(z_1, z_2, z_3, n_w, m_n, b)$ satisfying the functional constraints is first determined. This is shown in Fig 3. It can be seen that distribution of number of solutions regarding gear ratio is approximately normal (Gauss) distribution. The same holds for the gear transmissions applied in the industry.
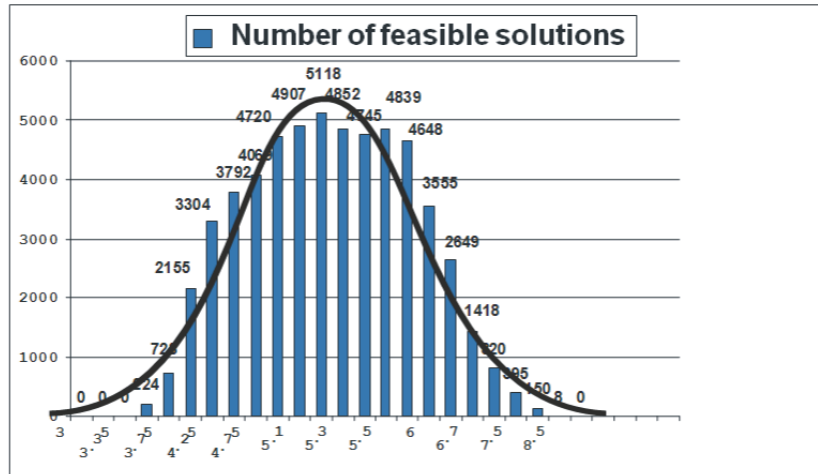
Fig.3 Numbers of feasible solutions

Next, the input data is chosen for examples of optimization methods application: $i = 5.3$, $n_{in} = 1000\,\text{min}^{-1}$, $T_{in} = 520\,\text{Nm}$, $L = 8000\,\text{h}$, $K_A = 1.25$, $IT7$ for all gears, material z_1 /material z_2/material $z_3 = 20\text{MoCr4}/20\text{MoCr4}/34\text{CrNiMo6}$, $S_{H\,min} = 1.1$, $S_{F\,min} = 1.2$, $\Delta i = 4\%$, $z_1 = 15 \div 30$,;,.

The number of Pareto solutions is 45. The ideal values of functions are determined and shown in Table 1.

Table 1 *Ideal point coordinates*

f_1^{id} in mm³	f_2^{id} in kg	f_3^{id}	f_4^{id} in min
1247030.66	6.61	0.992	151.825

The Euclidean distance method with an ideal point as a reference point gives the solution, from Pareto set, shown in Table 2, with a set of objective functions values shown in Table 3.

Table 2 *Solution obtained by Euclidean distance method*

Solution	Variable values					
	$x_1 = z_1$	$x_2 = z_2$	$x_3 = z_3$	$x_4 = n_w$	$x_5 = m_n$	$x_6 = b$
A2911	29	48	-124	3	2.25	29

Table 3 *Objective function for solution A2911*

Solution	f_1 in mm³	f_2 in kg	f_3	f_4 in min
A2911	1793746.05	8.432	0.991	190.564

Taking into consideration the fact that it is necessary to determine the priority of functions for the next methods anticipated here, the following examples are noted.

Example 1. Let the function f_1 be the priority function. The obtained solution is given in Table 4, with the set of objective functions in Table 5. This solution is obtained by all of the methods: the weighted coefficients method, the lexicographic method and the ε-constraints method.

Table 4 *Solution obtained by prioritizing the function* f_1

Solution	Variable values					
	$x_1 = z_1$	$x_2 = z_2$	$x_3 = z_3$	$x_4 = n_w$	$x_5 = m_n$	$x_6 = b$
A1197	23	37	-97	4	2.5	27

Table 5 *Objective function for solution A1197*

Solution	f_1 in mm^3	f_2 in kg	f_3	f_4 in min
A1197	1247030.66	6.6105	0.989	184.7404

Example 2. Prioritizing the function f_2, all applied methods also indicated the solution A1197.

Example 3. Prioritizing the function f_3, all applied methods point to the solution A3240, Table 6 and Table 7.

Table 6 *Solution obtained by prioritizing the function* f_3

Solution	Variable values					
	$x_1 = z_1$	$x_2 = z_2$	$x_3 = z_3$	$x_4 = n_w$	$x_5 = m_n$	$x_6 = b$
A3240	30	51	-130	4	2	28

Table 7 *Objective function for solution A3240*

Solution	f_1 in mm^3	f_2 in kg	f_3	f_4 in min
A3240	1524475.33	8.00	0.992	230.055

Solution A3240 is much closer to the solution obtained by the Euclidean distance method than the solution to which the priority of functions f_1 and f_2 point. The reason is the aforementioned fact that for obtaining the maximum of function f_3, a planetary gear train of huge dimensions is needed.

Example 4. Finally, function f_4 is assumed as the priority function. The weighted coefficients method and the ε - constraints method point to the solution A324, Table 8 and Table 9. The application of the lexicographic method gives solution A39, Tables 10 and 11.

Table 8 *Solution obtained by weighted coefficients method and by prioritizing the function* f_4

Solution	Variable values					
	$x_1 = z_1$	$x_2 = z_2$	$x_3 = z_3$	$x_4 = n_w$	$x_5 = m_n$	$x_6 = b$
A324	20	32	-85	3	3	31

Table 9 *Objective function for solution A324*

Solution	f_1 in mm^3	f_2 in kg	f_3	f_4 in min
A324	1563334.317	7.45	0.987	153.3

Table 10 *Solution obtained by lexicographic method by prioritizing the function* f_4

Solution	Variable values					
	$x_1 = z_1$	$x_2 = z_2$	$x_3 = z_3$	$x_4 = n_w$	$x_5 = m_n$	$x_6 = b$
A39	17	28	-73	3	3.5	32

Table 11 *Objective function for solution A39*

Solution	f_1 in mm^3	f_2 in kg	f_3	f_4 in min
A39	1651105.128	7.802	0.985	151.825

It is obvious that in this example the choice for optimal solution is between A39 and A324, and the solution A39 is adopted.

5. Comparison of optimization methods and program results analysis

Solutions obtained by these methods for multicriteria optimization are coordinated. The methods, although starting from different prepositions and having different mathematical bases, lead to harmonized results, which provide a physical meaning.

Particularly, the coordination between the weighted coefficients method and the lexicographic method can be easily observed. These two methods give identical results in three examples, but that is not the case in example 4. The weighted coefficients method reduces the problem to one single criterion optimization problem, and the lexicographic method requires solving three single optimization problems. When the fourth function has the priority, there are only few solutions in the first iteration, and they mainly have a small number of gear teeth. The priority of other functions has a very small influence on the final solution and hence is given consequently.

The ε-constraints method can be important if significant constraints are recognized. This method is especially applicable in situations when one function has the primary importance, while others must be in the range of some allowed limits. Setting the limits for functions has significant influence on the result. In these examples, the aim was to set the limits closer to ideal point in order to make the comparison of methods. This means that if constraints are not decisive, but are the priority of functions, some other methods should be applied.

The Euclidean distance method does not take into consideration preferences of particular criteria. This method is very suitable in the case when there is some referent solution which is usually infeasible. By applying this method, the solution closest to this referent solution is obtained. A good choice for the referent solution is an ideal point.

It can be concluded that, in the case of the existence of priority functions it is suitable to give precedence to the weighted coefficients method due to very clear physical meaning and experience in application to technical systems optimization [4,6]. It is also possible to apply this method in the case of equal priority functions.

Results of the computer program can be shown by diagrams- criterion space. Based on all of them, the cause-effect relations between particular objective functions given as their absolute values can be determined. The next figures show the following characteristic criterion spaces. It can be observed that functions f_1 and f_2 $(-V,-m)$, Fig. 4, have a strong correlation, almost a linear one. It is in accordance with the nature of these functions and the established mathematical model. The dependence of the criteria f_2 and f_3, $(-m,\eta)$, Fig. 5, consists of the family of lines parallel to the $x-$axis. It can be seen that f_2 and f_3 are not correlated. Efficiency values f_3 vary in the short range, since in this model efficiency is defined only as a function of teeth numbers. The dependence of criteria f_1 and f_3, $(-V,\eta)$, Fig. 6, has the same shape in the criteria space as the dependence in Fig.5. This is in accordance with the first conclusion of a very strong correlation between criteria f_1 and f_2. Also, the same shape has the dependence of criteria f_3 and f_4, $(\eta,-T)$, Fig. 7. That can be explained by the direct dependence of function f_4 from functions f_1 and f_2, shown in next

figures, Fig. 8 and Fig 9. The dependence of criteria f_2 and f_4, $\left(-m,-T\right)$, Fig. 8, is the direct dependence, but weaker than the dependence between f_1 and f_2. Some dispersion in the y-axis direction can be observed. This dependence is also justified and expected, because the production costs naturally depend on mass ($T = f(m)$). It is not linear, since f_4 does not only include the gear mass but also the other production costs, which are expressed by very complicated relations. However, the dependence can be described by the set of almost parallel lines. The dependence of criteria f_1 and f_4, $\left(-V,-T\right)$ has the same shape, Fig. 9.
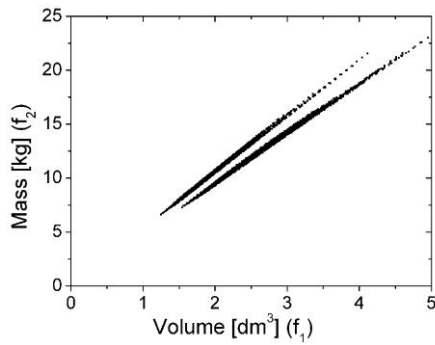
Fig. 4 Dependence of the criteria f_1 and f_2 **Fig. 5** Dependence of the criteria f_2 and f_3
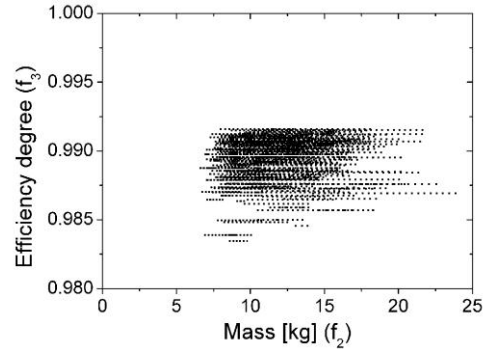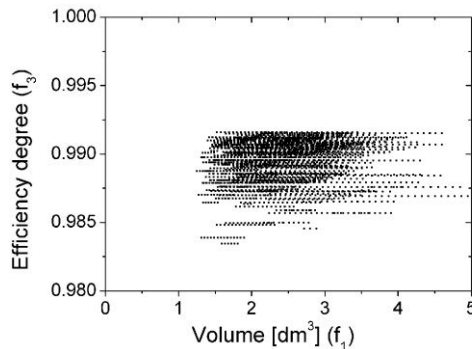

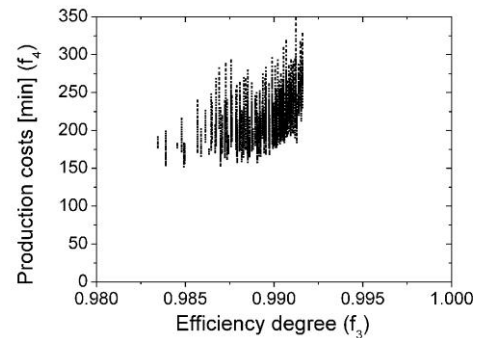**Fig. 6** Dependence of the criteria f_1 and f_3 **Fig. 7** Dependence of the criteria f_3 and f_4
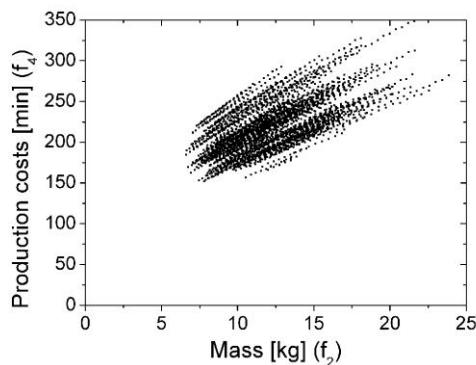

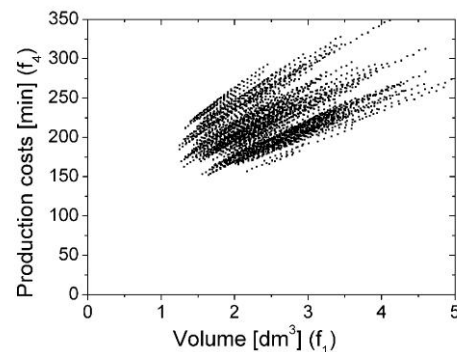**Fig. 8** Dependence of the criteria f_2 and f_4 **Fig. 9** Dependence of the criteria f_1 and f_4

It can be concluded that functions f_1 and f_2 have a strong correlation, function f_4 is weakly correlated with f_1 and f_2, while function f_3 has no dependences.

Prioritizing the function f_1 or f_2 in this numerical example indicates the same solution (PGT determined by design parameters) with the same set of objective functions. The application of this procedure and other examples, i.e. with different input data, usually yields the same conclusion but that is not always the case. Some examples are close, but they do not give the same values. Therefore, it should not be concluded that it is sufficient to use only one function. It is more appropriate to use both functions for further optimization.

6. Conclusion

In this paper, an original model for multicriteria optimization of planetary gear trains is presented. The complete optimization procedure based on an original algorithm is implemented in the newly developed *PlanGears* software. The basic type of planetary gear train is the subject of the paper. The mathematical model consists of objective functions, variables and functional constraints. Besides the determination of the set of Pareto optimal solutions, the presented original approach includes methods which select an optimal solution for the input data from the Pareto solutions set. There are: weighted coefficients method, lexicographic method, the ε - constraints method and distance method.

Based on numerical examples obtained by application of this software, the comparison of the optimization methods and program results analysis are presented. Although the methods start from different prepositions and have different mathematical basis, they lead to harmonized results, which provide them a physical meaning. As an illustration of program possibilities, a graphical review as diagram-criteria space is also shown. From it, the mutual dependence and connections between objective functions are pointed to. The connections between objective functions obtained from diagram-criteria space are in accordance with the nature of the functions and the established model.

This approach is original in planetary gear train optimization and can be successfully used for the basic planetary gear train type. Results obtained in this way are in accordance with the literature on technical system optimization and indicate a good choice of applied methods. Furthermore, this approach indicates a possibility for application to other planetary gear train types.

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